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INFLUENCE OF THE FRICTION COEFFICIENT ON THE SEISMIC BEHAVIOR OF INCA STONE MASONRY

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SUMMARY

One notable characteristic of the Inca stone masonry is that the finely carved stone blocks were fitted without any kind of mortar. However, most of these historical monuments are located in the west side of South America, which is highly seismic zone. The understanding of the dynamic behavior of this type of masonry is important for its preservation. The shapes of the blocks, as well as, the frictional forces generated between these stone blocks without mortar, would generate a particular mechanism against lateral forces from earthquakes. This research attempts to explain some of the mechanisms of Inca stone construction through experiments on roughness and on friction coefficient of stone blocks. In-situ measurements of the roughness and experimental measurements of the friction coefficients were conducted. A series of shaking table test on small scale simple model of stone blocks or bricks were performed. Likewise, a mathematical model was developed to estimate the dynamic behavior of simple stone bricks under seismic loads. The theoretical model takes into account the friction forces, as well as, the possible impact forces between stone bricks during shaking. According to the results from mathematical model, a uniform friction would be appropriate to reduce the possible impact forces. However, a high dispersion of the value of roughness (friction coefficient) for the same brick was obtained from experiments.

1. INTRODUCTION

It is well known that the Incas used mainly stone masonry for important buildings located in the Andean region. These types of constructions can be still appreciated in Cusco city, Machu Picchu, Choquequirao, and other Inca settlements. Since these constructions are located in a zone of high seismic activity like the western part of South America, the understanding of the seismic behaviour of this historical building become important task for management and preservation activities.

Three principal styles of Inca stone masonry can be distinguished. The first style is a roughly shaped stone masonry which units of irregular shape were fitted with mud mortar, as is shown in Figure 1(a). The second style is a finely shaped masonry with polygonal units fitted without mortar, as is shown in Figure 1(b). The third style

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is a finely shaped masonry of rectangular blocks forming horizontal layers as is shown in Figure 1(c). In this research, the third style was chosen to study the influence of the friction coefficient in the seismic behaviour of the masonry, since it is believed that the blocks work under the friction force among blocks piled up and under the impact forces between lateral blocks that could be developed during earthquakes.

(a) Roughly shaped stone masonry (b) Polygonal masonry (c) Rectangular masonry

Figure 1: Types of Inca stone masonry

To estimate the friction coefficient, the measurements in-situ were performed in selected building of Machu Picchu citadel and in the Coricancha Temple in Cusco. To define the characteristics of the model to be used in the shaking table test, two surface finishing methods that are used at present in Japan, were employed to make replicas with similar properties to Inca stone.

2. ESTIMATION OF THE FRICTION COEFFICIENT

Two methods were employed in-situ to estimate the friction coefficient of Inca stone blocks. First, replicas of the surface were taken by using industrial clay. Then, direct measurements of the surface roughness were performed by using a roughness meter. The selected places for these measurements were the Main Temple and the Temple of the Three Windows in Machu Picchu, and the Temple of Stars in Coricancha Complex (Cusco city).
The selected places in Machu Picchu are shown in Figure 2. Figure 3 shows the Temple of Stars in Coricancha as well as a detail when one replica is taken with industrial clay.

Figure 2: Main Temple and temple of Three Windows in Machu Picchu

The replicas of horizontal surfaces and vertical surfaces were obtained from selected locations in the respective construction. To make one replica, the surfaces with dust were cleaned up, and later, a portion of industrial clay was warmed to an appropriate temperature (50^{o}C approximately) to obtain adequate plasticity, and was applied on the stone surface. This industrial clay was retired from the surface after getting its solid state, and sent to laboratory. The replicas were analyzed using a tri-dimensional surface analyzer to obtain the roughness of the replica surface. This roughness is measured as the average of the maximum deformation with respect to the average surface deformation, which it is expressed in micrometers. The values of the estimated roughness are shown in Table 1.

Figure 3: Temple of Stars in Coricancha and a detail when one replica is taken

Table 1: Stone roughness estimated from replica samples

Sample	Machu Pichu (μm)	Coricancha (μm)
1	43.26	26.59
2	31.89	51.03
3	32.96	26.70
4	41.83	37.38
5	25.87	27.03
6	36.27	26.45
average	35.34	32.53

Table 2: Stone friction coefficient estimated in-situ measurements

Sample	Machu Pichu	Coricancha
1	0.57	0.43
2	0.53	0.39
3	0.41	0.45
4	0.42	0.50
5	0.39	--
average	0.46	0.44

The average stone friction coefficients (5 points in Machu Picchu and 4 points in Coricancha) obtained with a portable friction coefficient meter, are shown in Table 2. Test specimens were constructed using the above parameters obtained from stone surface finishing methods. The types of finishing used in this research are: bush hammered, dabbed finished, rough grind, rubbing and polishing. The roughness values and friction coefficients for each type of finishing are displayed in Table 3.

Table 3: Roughness and friction coefficient for different types of finishing

Type of Finishing	Roughness (μm)	Friction Coefficient
Bush hammered	66.79	0.63
Dabbed finished	30.75	0.48
Rough grind	22.14	0.53
Rubbing	5.68	0.50
Polishing	1.28	0.39

From these results, it can be observed that the dubbed finished type with 30.74μm of roughness and 0.48 of friction coefficient could be a representative finishing of the Inca stone. In consequence, this finishing type was selected to construct specimens for shaking table test.

3. SHAKING TABLE TEST OF STONE MODEL

To model the friction effect and the forces due to the impact, a simple model consisting of five blocks was selected for shaking table test. Figure 4 shows the model, which consists of three blocks of 100x50x100 mm and two blocks of 50x50x100 mm. A stone base with dabbed finished style was fixed on the shaking table, and the blocks were piled up on this base.

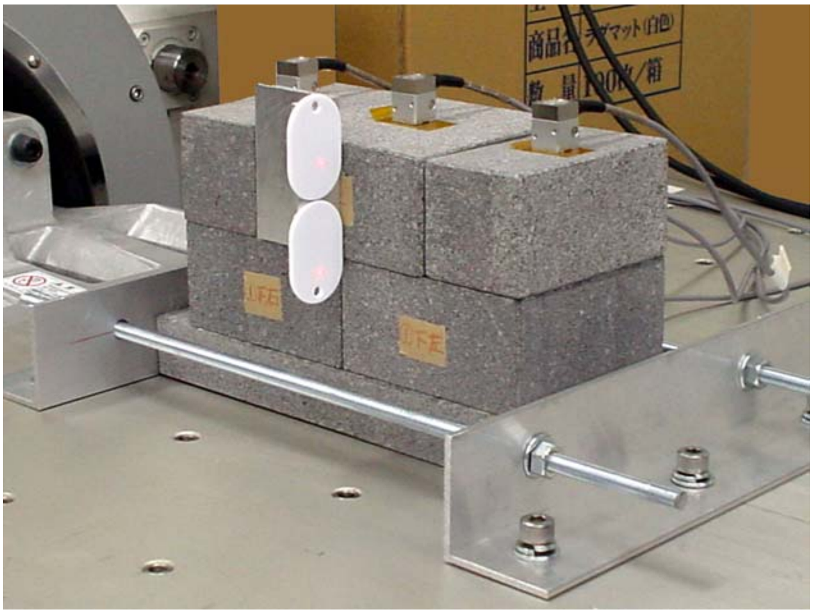

Figure 4: Stone model piled up on the shaking table

The signal recorded by the Japan Meteorological Agency during the 1995 Kobe Earthquake was employed as input motion. This input motion was applied 5 times successively. The remaining relative displacements between blocks were measured after finishing each run. Figure 5 shows the state of the model after finishing five runs.

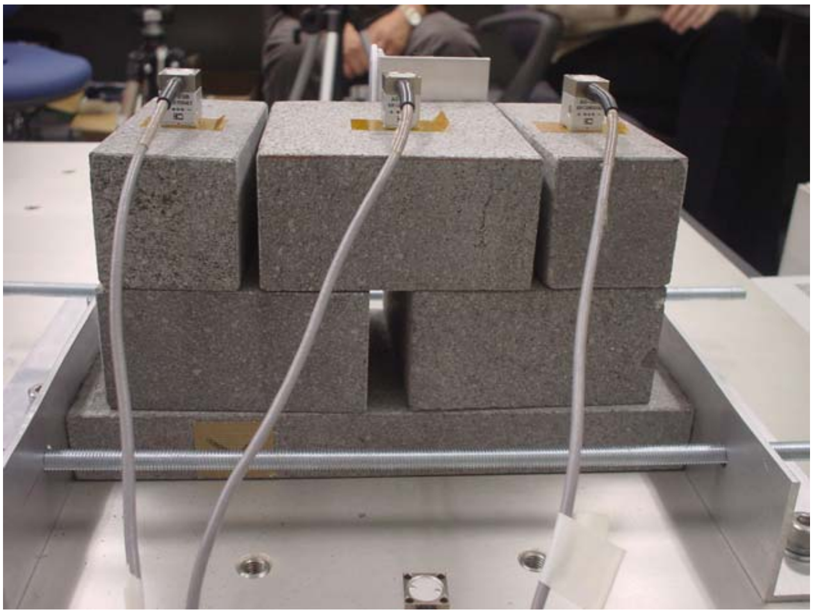

Figure 5: Final state of the model after five runs of the Kobe Earthquake (JMA)

4. ANALYTICAL MODEL

To develop a numerical model, the blocks were labeled as is shown in Figure 6. This model considers friction coefficient or frictional forces acting in horizontal joints between blocks. Likewise, the impact forces were considered in vertical joints between adjacent blocks.

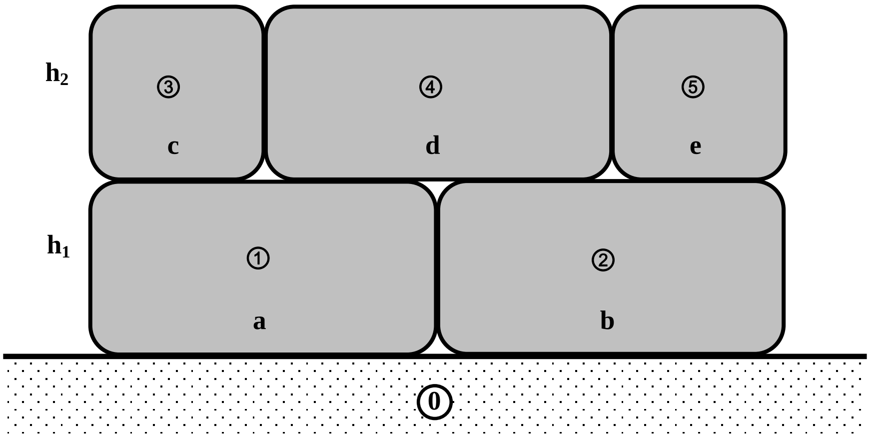

Figure 6: Multiple rigid body model

The equation of motion for this system can be expressed as follows:

$$[M]\{a\}+\{f\}=-\{m\}a_g \tag{1}$$

where $[M]$ is the mass matrix, $\{a\}$ is the acceleration vector, {f} is the force vector due to the friction between blocks, and a_g is the input acceleration at the ground. The mass matrix is a diagonal matrix with the masses of each block as elements.

The force vector due to the friction is evaluated considering the equilibrium of dynamic forces in the multiple rigid body system, which is giving by the following equation:

$$\{f\}=\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{Bmatrix}=\begin{Bmatrix} F_{10}\,\mathrm{sgn}\,v_1 - F_{31}\,\mathrm{sgn}(v_3-v_1)-F_{41}\,\mathrm{sgn}(v_4-v_1) \\ F_{20}\,\mathrm{sgn}\,v_2 - F_{42}\,\mathrm{sgn}(v_4-v_2)-F_{52}\,\mathrm{sgn}(v_5-v_2) \\ F_{31}\,\mathrm{sgn}(v_3-v_1) \\ F_{41}\,\mathrm{sgn}(v_4-v_1)+F_{42}\,\mathrm{sgn}(v_4-v_2) \\ F_{52}\,\mathrm{sgn}(v_5-v_2) \end{Bmatrix} \tag{2}$$

where, the F_{ij} represents the friction force between the upper block i and the lower block j. In the analysis, it is considered that the forces become active only in the case that the input acceleration is larger that the product of the friction factor and the gravity acceleration. That is, the friction force acts in the dynamic equation only when the input force is larger that the static friction force. Furthermore, the sign of the friction forces depends on the sign of the relative velocities between blocks. These friction forces F_{ij} are calculated by considering the weight of the blocks and the assumed friction coefficient μ. The weight of each block is calculated from the dimension of blocks and for a given specific weight of the material γ (in this case stone material). Therefore, the following expressions are used to estimate these frictional forces:

$$\begin{aligned} F_{10} &= [at(h_1+h_2)\gamma]\mu_{10} \\ F_{20} &= [bt(h_1+h_2)\gamma]\mu_{20} \\ F_{31} &= [cth_2\gamma]\mu_{31} \\ F_{41} &= [(a-c)th_2\gamma]\mu_{41} \\ F_{42} &= [(b-e)th_2\gamma]\mu_{42} \\ F_{52} &= [eth_2\gamma]\mu_{52} \end{aligned} \tag{3}$$

The equation of motion (Eq. 1) is solved numerically by means of the Newmark integration scheme. In this case, the expression of the equation of motion for the step i+1 is given by:

$$[M]\{a\}_{i+1} + \{f\}_{i+1} = -\{m\}a_{g_{i+1}} \tag{4}$$

Then, the following equations for displacement, velocity and acceleration are used in an iterative scheme:

$$\begin{aligned} \{d\}_{i+1} &= \{d\}_i + \Delta t\{v\}_i + \frac{\Delta t^2}{2}\{a\}_i \\ \{v\}_{i+1} &= \{v\}_i + \frac{\Delta t}{2}\left(\{a\}_i + \{a\}_{i+1}\right) \\ \{a\}_{i+1} &= [M]^{-1}\left[-\{m\}a_{g_{i+1}} - \{f\}_{i+1}\right] \end{aligned} \tag{5}$$

In addition to the friction forces, the effect of impact forces between adjacent blocks is considered in the equation of motion. For this instant, an elastic impact is assumed and therefore the coefficient of restitution is equal to 1.

5. RESULTS

The residual displacement of each block after five runs of the Kobe Earthquake as input motion is shown in Figure 7. Each set of input motion was called run1, run2, run3, run4 and run5 respectively. All runs were supposed of having same conditions. However, the final displacement differs for each block. In spite of that, the general pattern with large displacements of blocks 3 and 4 and smaller displacements for block 2 were obtained in all runs.

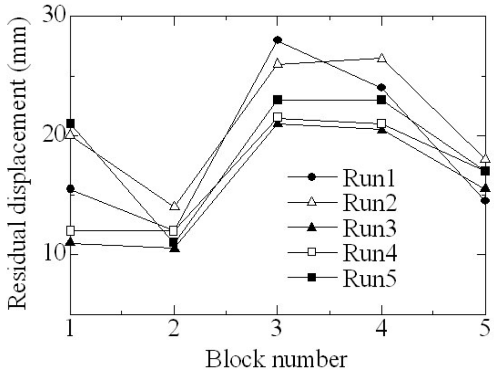

Figure 7: Experimental results: final displacement after a set of Kobe earthquake input motion

In this analysis, the friction coefficients were first selected randomly around 0.4 with a variation of ±0.02. Results of this analysis are shown in Figure 8. The order of the final displacements around 20 mm agreed with those of the experimental results. However, the general pattern is completely different with respect to the experimental results, with larger displacements in blocks 1 and 2.

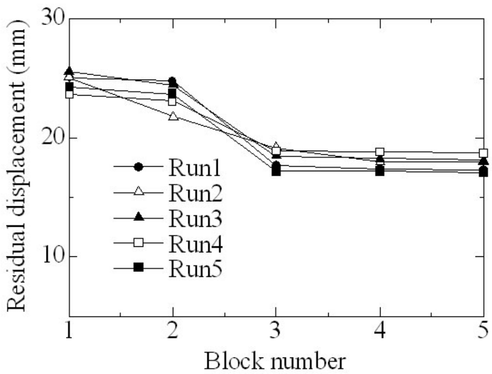

Figure 8: Analytical results with friction coefficient values around 0.4

The use of almost uniform friction coefficient in the analysis gave as result that the displacement pattern of each block differs from the one obtained experimentally. Therefore, a series of sets of friction coefficients with larger variation and randomly selected were used for a new analysis. Approximately 50 sets were tested and it was observed that the variation of the friction coefficient has an important influence in the results. From these trials, 3 runs are selected and their results are presented in Figure 9. The friction coefficients used in the last analyses are shown in Table 4. As can be observed from Figure 9, it is possible to get a similar pattern of the final state of the blocks compared with experimental results, by controlling or changing the values of friction coefficients. It can be noted that larger friction coefficients in the contact of lower blocks and base produce results that agree better with experimental results. Probably, in experimental cases, the friction at the base is larger than the friction between blocks since these elements were prepared separately, as well as, the type of polishing of the blocks and the base are different.

Table 4: Friction coefficients used in analysis with large variation of the average value

Set	μ_{10}	μ_{20}	μ_{31}	μ_{41}	μ_{42}	μ_{52}
Run1	0.40	0.50	0.30	0.44	0.44	0.31
Run2	0.47	0.30	0.50	0.39	0.32	0.35
Run3	0.43	0.51	0.52	0.37	0.29	0.45

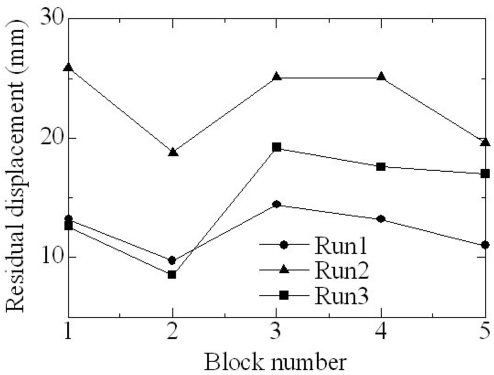

Figure 9: Analytical results with large variation in the friction coefficient

6. CONCLUSIONS

An evaluation of the friction coefficient of the Inca stone masonry block was performed. The average values between 0.39 and 0.57 were obtained from in-situ measurements. These characteristics of friction and roughness agreed well with a present technique used for stone polishing named dabbed finishing.

Experimental small scale model was subjected to a series of input motion to simulate its seismic behavior. Also analytical model of multiple rigid bodies was employed to compare with experimental results. The analytical model takes into account the friction between stone blocks and the action of impact forces between adjacent blocks.

Considering almost uniform friction among blocks, the final state of blocks differs from that the one obtained experimentally. However, a random large variation of these coefficients could reproduce the experimental behavior.

The research has confirmed the strong influence of friction coefficient in the response of a simple model of stone masonry. A variation of this factor gives different response patterns.

These findings are part of the ongoing collaborative research initiative at Akita Prefectural University and more sophisticated and detailed investigations, which include the effect of the shape of blocks, are currently underway.

7. REFERENCES

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